HEAT TRANSFER BETWEEN A STREAM OF LIQUID AND A POLYDISPERSION IN A FLUIDIZED BED

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An approximate formula is derived for the heating time of particles in a fluidized polydispersion bed.

Knowing the time in which particles will heat up to a given temperature is important in many chemical engineering processes, in drying, and in heat treatment in intermittently operating fluidization beds. While this problem can be solved easily in the case of a monodispersed material, and has been already solved with various degrees of approximation [1-4], it is difficult to solve when the solid phase of a fluidized bed contains a polydispersed material and the heat transfer particles of different sizes and at different temperatures must be taken into account. The problem is solved, to the first approximation, by reducing it to the problem of heating a monodispersion bed whose particles are of an equivalent diameter [1, 2]. Many effects related to polydispersivity and often of crucial importance remain then neglected [5, 6].

We consider a homogeneous isothermal fluidized polydispersion bed in a reactor of uniform cross section and with a given size distribution φ (R) of spherical particles for which the following normalization applies:

$$\int\limits_{R_1}^{R_2} \varphi(R)\, dR = N. \tag{1}$$

Let the heat transfer between particles and the medium begin after a hot fluidizing agent or a fresh charge of cold material has been added. We will then assume that the temperature of the medium remains at this time constant and equal to the entrance temperature. The conditions under which this premise is valid follow directly from an analysis of the equations [1-3]:

$$\begin{aligned} c_G\rho_G v_G(t_G - t_{0G}) &= \alpha_1 4\pi R^2 hN(t_S - t_G), \\ c_G\rho_G v_G(t_G - t_{0G})\, d\tau &= c_S\rho_S \frac{4}{3}\pi R^3 hN dt_S. \end{aligned} \tag{2}$$

Indeed, if $\alpha_1 4\pi R^2 hN/c_G\rho_G v_G \approx 0$ or $c_S\rho_S 4\pi R^3 hN/3c_G\rho_G v_G \approx 0$, then $t_G \approx t_{0G}$.

In subsequent calculations this condition will be assumed satisfied. In the course of heating up, particles with radius R will receive heat not only from the fluidizing agent but also from other particles at higher temperatures, while they will transmit heat to particles at lower temperatures through their contact with them.

We will assume further that particles in a fluidized bed are heated by the following mechanism, which was first mentioned in [3]. The packet of particles heated to a reference temperature σ near the temperature of the medium comprises with it a homogeneous diluted bed containing also particles outside that packet at a lower temperature. In the course of heating, obviously, the number of elements in the fluidized packet of particles will increase.

Noting that Bi $\to 0$ for the particles and that particles with smaller diameters heat up faster, we write the equation of heat transfer between particles with radius R and the fluidized bed:

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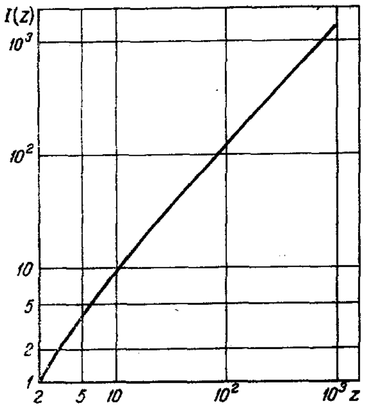

Fig. 1. Graph of I = f(z).

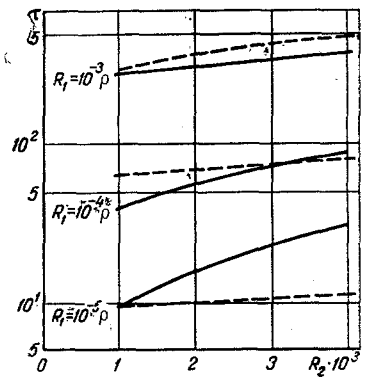

Fig. 2. Results of calculations by formula (18) (solid curves) and by formula (24) (dashed curves). Time τ (sec), radius R (m).

$$\frac{d\Theta_S(R,\ \tau)}{d\tau} = -\frac{3\alpha(R_\tau,\ R)}{c_S\rho_S R}\,\Theta_S(R,\ \tau). \tag{3}$$

An analogous equation has been derived in [5, 6] for a throughfeed flow, but for a fluidized bed it simplifies, because the mean velocities of the various fractions must all be considered equal to zero.

Equation (3) will now be rewritten as

$$\frac{d\ln\Theta_S(R,\ \tau)}{d\tau} = -\frac{3\alpha(R_\tau,\ R)}{c_S\rho_S R}. \tag{4}$$

Since R_τ is a function of time, we may write $\alpha(R_\tau,\ R) \equiv \alpha(\tau,\ R)$ and the solution to (4) will be

$$\ln\Theta_S(R,\ \tau) = -\frac{3}{c_S\rho_S R}\int_{\tau_1}^{\tau}\alpha(\tau',\ R)\,d\tau' + \ln\Theta_S(R,\ \tau_1). \tag{5}$$

Here $\Theta_S(R,\ \tau_1)$ is the temperature of particles with the radius R at the instant of time τ_1, when the particles with radius R_1 have reached temperature σ. Temperature $\Theta_S(R,\ \tau_1)$ can be found analogously to (4), if $\alpha(R_\tau,\ R)$ is replaced by the coefficient of heat transfer between these particles and a pure fluidizing agent – this coefficient being assumed independent of time:

$$\frac{d\ln\Theta_S(R,\ \tau)}{d\tau} = -\frac{3\alpha_1(R)}{c_S\rho_S R}. \tag{6}$$

Then, with

$$\Theta_S(R,\ 0) = 1,\quad \Theta_S(R_1,\ \tau_1) = \sigma \tag{7}$$

taken into account, we have

$$\ln\Theta_S(R,\ \tau_1) = -\frac{3\alpha_1(R)}{c_S\rho_S R}\,\tau_1 \tag{8}$$

and

$$\tau_1 = \frac{c_S\rho_S R_1}{3\alpha_1(R_1)}\ln\frac{1}{\sigma}. \tag{9}$$

Letting $\Theta_S(R,\ \tau) = \sigma$ in (5), we find the time in which particles with radius R heat up to temperature σ. Since the heating time τ will in this case depend on the radius R, one may write

$$\alpha[\tau(R),\ R] \equiv \alpha(R);\quad d\tau = \frac{d\tau}{dR}\,dR. \tag{10}$$

Then (8), (9), and (10) yield

$$\ln\sigma = -\frac{3}{c_S\rho_S R}\int_{R_1}^{R}\alpha(R')\frac{d\tau}{dR'}\,dR' - \frac{\alpha_1(R)}{\alpha_1(R_1)}\cdot\frac{R_1}{R}\ln\frac{1}{\sigma} \tag{11}$$

or

$$\int_{R_1}^{R}\alpha(R')\frac{d\tau}{dR'}\,dR' = \frac{c_S\rho_S}{3}\ln\frac{1}{\sigma}\left[R - \frac{R_1}{\alpha_1(R_1)}\alpha_1(R)\right]. \tag{12}$$

Differentiating (12) with respect to R, we obtain

$$\alpha(R)\frac{d\tau}{dR} = \frac{c_S\rho_S}{3}\ln\frac{1}{\sigma}\left[1 - \frac{R_1}{\alpha_1(R_1)}\cdot\frac{d\alpha_1(R)}{dR}\right]. \tag{13}$$

From here

$$\tau(R) = \tau_1 + \frac{c_S\rho_S}{3} \ln \frac{1}{\sigma} \int_{R_1}^{R} \left[1 - \frac{R_1}{\alpha_1(R_1)} \cdot \frac{d\alpha_1(R')}{dR'} \right] \frac{dR'}{\alpha(R')} . \tag{14}$$

The empirically found relation between the heat transfer coefficient and the particle dimensions is often approximated by a power law:

$$\alpha_1(R) = aR^m; \quad \alpha(R_{equ}) = bR^n_{equ}. \tag{15}$$

If the size distribution of particles is taken to follow the Rosine–Rammler relation [7]

$$\varphi(R) = c \exp\left[-\left(\frac{R}{R_0} \right)^p \right] \tag{16}$$

with the polydispersivity index p = 1 and $R_0 \gg R_1$, $R_0 \gg R_2$, then

$$R_{equ} = \sqrt{2} R_1 R \sqrt{\frac{\ln R/R_1}{R^2 - R_1^2}} . \tag{17}$$

With the aid of (15) and (17), (14) yields

$$\tau(R) = \frac{c_S\rho_S R_1}{3} \ln \frac{1}{\sigma} \left\{ \frac{1}{aR_1^m} + \frac{(\sqrt{2} R_1)^{-n}}{b} I(z) \right\} , \tag{18}$$

where

$$z = R/R_1;$$

$$I(z) = \int_1^z (1 - m\xi^{m-1}) \left(\frac{\xi^2 \ln \xi}{\xi^2 - 1} \right)^{-\frac{n}{2}} d\xi. \tag{19}$$

For calculations we use empirical relations for the coefficient of heat transfer between particles and pure gas [3]

$$\mathrm{Nu} = 0{,}12 \mathrm{Re}^{1{,}03} \mathrm{Pr}^{0{,}54} \tag{20}$$

or between a fluidized bed and an immersed surface [2]

$$\alpha = 29{,}5 \rho_S^{0{,}2} \lambda_G^{0{,}6} R^{-0{,}36} . \tag{21}$$

For air at a 100°C temperature and flowing at a rate v_G = 0.2 m/sec as the fluidizing agent, and for corundum particles as the solid phase, we find (using system S1)

$$a = 24{,}3, \quad b = 18{,}5, \quad m = 0{,}03, \quad n = -0{,}36. \tag{22}$$

With the aid of a "Promin" computer, the integral (13) has been tabulated for chosen values of m and n with z ranging from 1 to 1000.

The graph I = f(z) is shown in Fig. 1. This function can be approximated by the polynomial

$$\lg I(z) = -0{,}02 \lg^5 z + 0{,}07 \lg^4 z + 0{,}30 \lg^3 z - 1{,}68 \lg^2 z + 3{,}55 \lg z - 1{,}18 \tag{23}$$

at z values not very close to unity.

The graphs in Fig. 2 are based on formula (18) and on the approximate formula

$$\tau = \frac{c_S\rho_S R_{equ}^{1-m}}{3a} \ln \frac{1}{\sigma} , \tag{24}$$

respectively, where R_{equ} has been determined from (17). Here $\sigma = 10^{-2}$.

It is evident from the graphs that formulas (18) and (24) may differ either way and the difference may be considerable especially for high values of the ratio R_2/R_1.

When particles are distributed in discrete sizes, then the integration in (14) should be replaced by a summation and

$$\tau(R_M) = \frac{c_S\rho_S R_1}{3} \ln \frac{1}{\sigma} \left\{ \frac{1}{\alpha_1(R_1)} + \sum_{i=2}^{M} \left[\frac{R_i - R_{i-1}}{R_1} - \frac{\alpha_1(R_i) - \alpha_1(R_{i-1})}{\alpha_1(R_1)} \right] \frac{1}{\alpha(R_{i-1})} \right\}. \quad (25)$$

If a fluidized bed comprises a mixture of particles of two sizes (M = 2), then

$$\tau(R_2) = \frac{c_S\rho_S R_1}{3} \ln \frac{1}{\sigma} \left\{ \frac{1}{\alpha_1(R_1)} + \left(\frac{R_2}{R_1} - \frac{\alpha_1(R_2)}{\alpha_1(R_1)} \right) \frac{1}{\alpha(R_1)} \right\}. \quad (26)$$

An analogous calculation according to this formula yields results which differ considerably (by up to 100%) from those obtained by the approximate formula (24).

NOTATION

R_1, R_2	is the minimum and maximum radius of particles respectively, m;
N	is the concentration of particles, m^{-3};
c_G, c_S	is the specific heat of the medium and of the particles respectively, J/kg • °C;
ρ_G, ρ_S	is the density of the medium and of the particle material respectively, kg/m^3;
τ	is the time, sec;
R_τ	is the maximum radius of particles heated up to the reference temperature σ, m;
$\alpha_1(R)$	is the coefficient of heat transfer between particles and fluidizing agent, W/m^2 • °C;
$\alpha(R_\tau, R)$	is the coefficient of heat transfer between fluidized bed and immersed surface, W/m^2 • °C;
t_G	is the temperature of the medium, °C;
t_S	is the initial temperature, °C;
t_{0S}	is the initial temperature of particles, °C;
$\Theta_S = (t_S - t_{0G})/(t_{0S} - t_{0G})$	is the relative temperature of particles;
R_{equ}	is the equivalent radius of a particle packet in the fluidized bed, m;
a, b, c	are the dimensional constants;
v_G	is the velocity of medium, m/sec;
h	is the height of bed, m;
λ_G	is the thermal conductivity of medium, W/m • °C;
Nu	is the Nusselt number;
Re	is the Reynolds number;
Pr	is the Prandtl number;

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